Multiple Phase Flow

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Lecture Notes: Homogeneous Flow Model

Introduction

The **homogeneous flow model** is one of the simplest models used to describe multiphase flow systems. In this model, the different phases (e.g., gas and liquid) are assumed to be completely mixed and flow as a single-phase fluid with uniform properties. It provides a foundation for understanding multiphase flows and serves as a starting point for more complex models.

Key Assumptions

Uniform Flow:

- The velocity of all phases is the same ($u_g = u_l = u$).
- There is no slip between the phases.

Uniform Properties:

- The phases are completely mixed, resulting in uniform properties (e.g., density and viscosity).

No Interphase Interactions:

- No relative motion or momentum exchange between phases.

Steady-State Flow:

The flow does not change with time (optional assumption, depending on the problem).

1. Mass Conservation:

For a control volume in steady-state conditions:

$$\frac{\partial(\rho u)}{\partial z} = 0$$

Where:

- ρ: Mixture density
- u: Mixture velocity
- z: Flow direction

2. Momentum Conservation:

$$\frac{\partial}{\partial z}(\rho u^2) = -\frac{\partial p}{\partial z} - \rho g + \tau_w$$

Where:

- p: Pressure
- g: Gravitational acceleration

τ_w: Wall shear stress

3. Energy Conservation:

$$\frac{\partial}{\partial z}\left(\rho u h + \frac{1}{2}\rho u^3\right) = q$$

Where:

- h: Specific enthalpy
- q: Heat transfer per unit length

Mixture Properties

In the homogeneous flow model, mixture properties are defined as weighted averages of the individual phase properties based on volume fractions (α_g for gas and α_l for liquid):

Mixture Density ($\boldsymbol{\rho_m}$):

$$\rho_m = \alpha_g \rho_g + \alpha_l \rho_l$$

Where:

- ρ_g, ρ_l: Densities of gas and liquid phases
- α_g, α_l: Volume fractions ($\alpha_g + \alpha_l = 1$)

Mixture Viscosity (μ_m):

$$\mu_m = \alpha_g \mu_g + \alpha_l \mu_l$$

Where:

- μ_g, μ_l: Viscosities of gas and liquid phases

Mixture Velocity (u_m):

$$u_m = \frac{\dot{m}}{\rho_m A}$$

Where:

- $\dot{m}$: Total mass flow rate
- A: Cross-sectional area

Applications

Two-Phase Flow in Pipes:

- Used in pipelines transporting oil, gas, and water mixtures.
- Simplifies analysis of pressure drop and flow rates.

Nuclear Reactor Systems:

- Analyzing coolant flow in boiling water reactors.

Chemical Process Engineering:

- Modeling gas-liquid reactors and flow in distillation columns.

Cryogenic Systems:

- Studying two-phase flow of liquid and vapor in cryogenic pipelines.

Advantages

Simplicity:

- Easy to implement due to reduced complexity.
- Fewer equations and assumptions are needed compared to other models.

Computational Efficiency:

- Requires less computational power compared to slip or drift models.

Useful for Preliminary Design:

- Provides approximate results for initial engineering analysis.

Limitations

No Slip Consideration:

- Assumes no velocity difference between phases, which is unrealistic in many practical cases.

Limited Accuracy:

- Does not account for phase interactions, making it less reliable for predicting detailed flow behavior.

Inapplicable to Flow Regimes with Phase Separation:

- Cannot describe stratified, annular, or churn flows where phases are not well-mixed.

Neglects Interphase Forces:

- Forces such as drag, lift, and turbulent diffusion are ignored.

Comparison with Other Models

Aspect	Homogeneous Model	Slip Models	Two-Fluid Models
Assumption	Single velocity field	Different velocities	Separate equations for phases
Complexity	Low	Moderate	High
Accuracy	Low	Moderate	High
Computational Cost	Low	Moderate	High

Example Calculation

Problem: A horizontal pipe carries a gas-liquid mixture with α_g=0.4, ρ_g=2 kg/m^3, ρ_l=1000 kg/m^3, and $\dot{m}$=50 kg/s. The pipe diameter is 0.1 m. Calculate the mixture density and velocity.

Solution:

Mixture Density:

$$\rho_m = \alpha_g \rho_g + \alpha_l \rho_l$$

$$\rho_m = (0.4)(2) + (0.6)(1000) = 600.8\ kg/m^3$$

Cross-sectional Area:

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.1)^2}{4} = 0.00785\ m^2$$

Mixture Velocity:

$$u_m = \frac{\dot{m}}{\rho_m A}$$

$$u_m = \frac{50}{600.8 \times 0.00785} \approx 10.6\ m/s$$

Problem Statement

A horizontal pipeline with a diameter of D = 0.1 m transports a gas-liquid mixture. The following properties are given:

- Gas volume fraction (αg) = 0.3
- Gas density (ρg) = 5 kg/m3
- Liquid density (ρl) = 1000 kg/m3
- Total mass flow rate ($\dot{m}$) = 100 kg/s

Determine the following:

Mixture density (ρm)

Mixture velocity (um)

Pressure drop per unit length ($\Delta P/L$) assuming a friction factor f = 0.02.

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Solution

$$\rho_m = \alpha_g \rho_g + (1 - \alpha_g)\rho_l$$

Substitute the given values:

$$\rho_m = (0.3)(5) + (1 - 0.3)(1000)$$

$$\rho_m = 1.5 + 700 = 701.5\,\text{kg/m}^3$$

The total mass flow rate is related to the mixture density and velocity by:

$$\dot{m} = \rho_m u_m A$$

Rearranging for um:

$$u_m = \frac{\dot{m}}{\rho_m A}$$

The cross-sectional area of the pipe is:

$$A = \frac{\pi D^2}{4}$$

$$A = \frac{\pi (0.1)^2}{4} = 0.00785\,\mathrm{m}^2$$

Substitute the values:

$$u_m = \frac{100}{701.5 \times 0.00785}$$

$$u_m \approx 18.19\,\mathrm{m/s}$$

Pressure Drop per Unit Length

The pressure drop per unit length in a pipe is given by the Darcy-Weisbach equation:

$$\frac{\Delta P}{L} = f\frac{\rho_m u_m^2}{2D}$$

Substitute the known values:

$$\frac{\Delta P}{L} = 0.02 \cdot \frac{701.5 \cdot (18.19)^2}{2 \cdot 0.1}$$

$$\frac{\Delta P}{L} = 0.02 \cdot \frac{701.5 \cdot 330.89}{0.2}$$

$$\frac{\Delta P}{L} = 0.02 \cdot 1151192.735$$

$$\frac{\Delta P}{L} \approx 23023.85\,\mathrm{Pa/m}$$

Conclusion

The homogeneous flow model is a simple and effective tool for approximating multiphase flow behavior in systems where phases are well-mixed and flow properties are relatively uniform. However, its limitations require engineers to supplement it with more advanced models for systems with significant phase separation or interphase interactions.